Logarithms

Time Doubling & Half-Life

In the automotive field, as in the real world, things double, but they also decay. Whether it be the amount of Horsepower an engine produces, the number of people living in a city or town, the speed of a computer processor or the number of germs growing in a bottle; or the deprecation value of a car, boat, or the amount aspirin decreases in our body after a few hours

Sometimes we have to use a mathematical process called a logarithm. A common logarithm is a logarithm to the base 10.

Or $\log_{10} 1000 = 3 \quad becasue\, 10^3 = 1000$

Vocabulary:

Logarithm: A number that represents a power or exponent. Thus, the common logarithm of 100 (log 100) is 2, because $10^2 = 100$ or…$\log_{10}$ means "10 to what power equals x ?"

Doubling time: The period of time required for a quantity to double in size or value (factor of 2)

Half-Life: The period of time required for a quantity to reduce by half in size or value (factor of 1/2)

Linear: Relating to, or resembling a line

Exponential: Expressed in terms of a designated power of e, the base of natural logarithms

Initial value (IV): The amount or value that we started with. *New value (NV)*: New amount or value

Quantity (Q): New amount or value. Q_0*: The amount or value that we started with.*

Rule of 70: A math formula to approximate the amount of time something will double or decay by half.

PDE Academic Standards:
2.4.11. D, 2.4.11. E, 2.5.11.A, 2.5.11.B, 2.5.11.C, 2.5.11.D, 2.6.11.A, 2.6.11.C: (Anchors: M11.E.1.1; M11.E.2.1; M11.E.4.1; M11.E.4.2;

Remember:

Pretty **P**lease **M**y **D**ear **A**unt **S**ally
(From left to right; **P**arentheses; **P**ower; **M**ultiply; **D**ivide; **A**dd, **S**ubtract)

Rule of 70: Same for growth or decay Example, Oil consumption is increasing at a rate of 4.2% a year. Approximate the time (in years) before oil consumption will double.

$$\frac{70}{P\%} \approx x \text{ years} \qquad \frac{70}{4.2\%} \approx 16.7 \text{ years}$$

Example 1: Using the rule of 70 to find out how long it will take the $100 I just put into my savings account a 5.5% interest to double ($200):

$$\frac{70}{5.5\%} \approx 12.7 \text{ years}$$

Let's check our work:

$$NV = IV(1+.055) = 200$$
$$NV = 100(1.055) = 200$$
$$\div both\ sides\ by\ 100$$
$$NV = \frac{Log\ 2.00}{Log\ 1.055} = 12.95 year$$

Example 2: Using the rule of 70 to find out how long it will take the $10,000 VW Beetle I just purchased to be worth half it's present value if it deprecates at 20% year:

$$\frac{70}{20\%} \approx 3.5 \text{years}$$

Let's check our work:

$$NV = IV(1-.20) = 5000$$
$$NV = 10,000(.80) = 5000$$
$$\div both\ sides\ by\ 10,000$$
$$NV = \frac{Log\ 0.50}{Log\ .80} = 3.10 years$$

Example 3: Doubling Exact Time: $NV = IV(2)^{t/Tdouble}$

Jon has 15,600 cars in his salvage (junk) yard. The number of vehicles in the yard doubles every 10 years, but he wants to become the largest salvager on the east coast. How many cars will Jon have in 18 years, 24 years and 35 years?

a. $NV = IV(2)^{18/10}$ b. $NV = IV(2)^{24/10}$ c. $NV = IV(2)^{35/10}$

Example 4: The population of a city doubles every 22 years. How long before it quadruples?

Example 5: The horsepower in a particular engine increase 5% every minute it is running. It has an initial 130 Hp. How long do you have to run the engine to reach its maximum 350 Hp?

$$IV \times (1+r)^t = NV$$

$$130Hp \times 1.05^t = 350Hp$$ In this case we **ADD** because the Hp is increasing

Divide both sides by 130

$$\frac{\log 2.69}{\log 1.05} = 20.28 \min$$

Example 6: New brake fluid contains 10,000 PPM of a particular anti-corrosion additive. If the additive breaks down 40% a year, how long before it reaches 1250 PPM and can longer protect the metal parts in the brake system?

$$IV \times (1 - r)^t = NV$$ In this case we **SUBTRACT** because the additive is decreasing

$$10,000 \times (1 - .40)^t = 1250$$

Divide both sides by 10,000

$$10,000 \times (.60)^t = 1250$$

$$\frac{\log .125}{\log 0.60} = 4.07 \text{ years}$$

Example 7: A particular brand of anti-freeze contains 75% its original Ethylene Glycol (EC). Ethylene Glycol has a half-life of 10 years. How old is the anti-freeze?

Half-life formula:

$$NV = \left(\frac{1}{2}\right)^{T/half}$$

$$NV = (.5)^{T/10}$$

$$= \log .5^{T/10} = .75$$

$$=^{T/10} \log .5 = \log .75$$

Multiple both sides by 10 to cancel $^{T/10}$

$$\frac{10 \log .75}{\log .50} = 4.15 \text{ years}$$

Example 8: In 1957, Chevrolet built 320,000 Belair's. These classics are now disappearing at a rate of 20% a year. In 2007 there were still 12,500 of these vehicles driving around. How many years before there are only 100 left on the road?

$$Q = Q_0 (1 - r)^t$$

$$= 12,500(1 - .20)^t = 100$$

$$= 12,500(.80)^t = 100$$

Divide both sides by 12,500

$$= \frac{\log .008}{\log .80} = 21.64 \text{ years}$$

Use the Rule of 70

As expected, as the number of classic 1957 Chevy's disappear, their value increases. If a 1957 perfect/showroom new condition Belair, has a current value of $75,000 if the value increases 12% a year, approximate how many years before it we will be worth $150,000? $\approx ?$

Exact time?

Example 9: $10^x = 30$ solve for x

Example 10: $14^x = 86$ solve for x

Example 11: $100^x = 30$ solve for x

Example 12: $\log 2x = 4$ solve for x

Example 13: $\log (2x) = 2$ solve for x

Example 14: $4 \times 10^x = 120$ solve for x

Exact time?

As expected, as the number of classic 1957 Chevy's disappear, their value increases. If a 1957 perfect/showroom new condition Belair, has a current value of $75,000 if the value increases 12% a year, approximate how many years before it we will be worth $150,000? $\approx ?$

Exact time?

$$T_{double} = \frac{\log 2}{\log(1+r)}$$

$$T_{double} = \frac{\log 2}{\log(1+.12)}$$

$$= \frac{.301}{.049}$$

$$= 6.14 \text{ years}$$